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Performance Analysis of Threshold Based Discrete Time Queuing System Using Early Arrival System

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Abstract: Queuing systems are crucial models for numerous practical scenarios where customers arrive, wait for service in a queue, and depart after receiving service. This study explores the dynamics of a threshold-based discrete queuing system with an early arrival system. The building block of the system includes an early arrival rate, a threshold, a service rate, and a few servers. The proposed research examines and evaluates the performance of a queuing system composed of two parallel queues and two series servers. Customers entering the system can opt for one of the two queues in parallel before transiting two consecutive servers in a series before departing the system. Such setup is used in a variety of fields, such as computer networks, customer service centers, and manufacturing processes. This research focuses on the major characteristics of the proposed queuing system, such as the mean number of customers, arrival and service rates, waiting times, flow times, and delays. These parameters offer information on system traffic, efficiency, and overall system performance.

Keywords: *Discrete Queuing System, Early Arrival System, Markov chain, Probability Mass Function, Threshold and delay.*

1. Introduction

Discrete-time queuing system is a method for digital network systems that provides flexibility and efficiency by means of slotted time. Discrete-time queuing systems have widespread applications in telecommunication systems. Discrete-time systems are more convenient to treat problems with its continuous counterpart [2]. In discrete model, time is divided into fixed-length intervals (called slots), which, in fact, play the role of the smallest undivided time units [6]. During a slot, a discrete-time queue can accept and serve only one customer at a time. The customer's arrival rate depends on server's state. Discrete-time queuing models are used to analyze and optimize various aspects of queuing systems. Service is synchronized with respect to slot boundaries, so packets cannot be serviced during their arrival slot [3].

The server takes a working vacation if the system is empty after service completion. During vacation, arriving customer will be served at a low rate. Server switches back to normal mode after vacation expires [1][5][6][12]. Sobolev *et al*. suggests that the number of customers in the system can be obtained from the normalized tail distribution [5].

The Threshold-Based Discrete-Time Queuing System with an Early Arrival System is a flexible and diverse mechanism in the domains of queuing theory and systems modeling, with applications in a wide range of fields. This system setup incorporates early arrivals and introduces the concept of thresholds to manage the commencement of queuing, adding a layer of complexity and adaptability to classic queuing models. By introducing thresholds, the Threshold-Based Discrete-Time Queuing System takes a revolutionary approach. These thresholds determine when the queuing system is activated, ensuring that entities are not executed unless an exact level of demand is satisfied. The concept provides benefits in cases when resources are expensive or scarce, allowing for more effective resource allocation.

Furthermore, the addition of an Early Arrival System increases the system's versatility. Entities coming prior to the system's official start can be treated differently, allowing for exclusive services, specialized treatment, or sophisticated routing tactics. This early arrival modifications represents real-world scenarios in which critical situations or pressing demands necessitate immediate assistance.

2. Methodology

2.1 System model

System as shown in figure 1, consists of two parallel finite queues and two servers in series where each queue has two internal thresholds to control the customer flow and servers provides service in two phases. Intensity of customers decides the service rate whether it remains normal or increases. This system handles two different types of customers with each type served by both phases.

In threshold-based discrete time queuing system, there is a limit on the number of customers allowed in the system (buffer or queue) before new arrivals are rejected or diverted. This threshold can be used to manage congestion and avoid delays. Early arrival system allows customers to enter the queue before the official start of the service time. When the

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system becomes operational, these arrivals are served immediately, impacting the overall system performance.

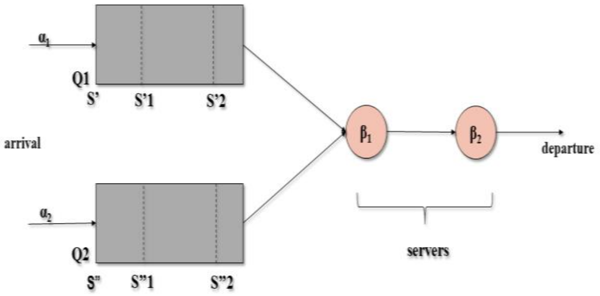

Figure 1: Threshold based Discrete Queuing Model

Here,
α = arrival rate
β = service rate
S = thresholds
Q = queue

2.2 Threshold Based Discrete Queuing Model

In the system, customers break into 2 arrivals and form 2 queues in parallel with thresholds that has finite buffer capacity. Customers entering the system have the choice of joining either of two parallel queues. Based on elements like service requirements or thresholds, this decision can be made.

Each parallel queue links to two series servers as shown in figure 2. Before leaving the system, entities must pass via both servers sequentially. The two servers can represent various processing phases.

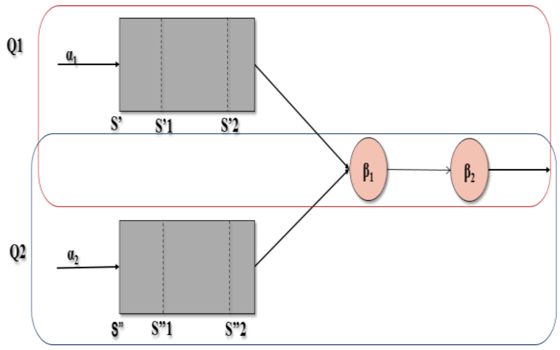

Figure 2: System Queuing Model for each queue

When customers enter the system, they select one of the two parallel queues. It queues up and wait for their service. The waiting time may vary depending on the particular queue they selected. Customers move via the first server in their respective series after exiting the queue. They then proceed to the second server before finally departing the system.

2.3 Markov Chain

Markov chains are a mathematical model used to represent a sequence of events in several domains, including probability theory, statistics, and computer science. When a Markov chain's collection of possible states is countable.

State Space:

A discrete Markov chain has a finite that can reflect various circumstances or arrangements.

Transition Probabilities:

There is a probability related to moving from one state to another for each pair of states. Typically, these probabilities are grouped in a transition matrix.

Memoryless Property:

The key feature of a Markov chain is that the probability of transitioning to a future state is controlled only by the present state and is independent of previous states.

The Markov chain illustrated in figure 3, for the system described have states like:
State 1: Arrival1 enters Queue1.
State 2: Arrival2 enters Queue2.
State 3: Server 1's Parallel Queue 1 is serving arrival1.
State 4: Server 1's Parallel Queue 2 is serving arrival2.
State 5: Server 2's Parallel Queue 1 is serving arrival1.
State 6: Server 2's Parallel Queue 2 is serving arrival2.

Process continues till reached final state where both arrivals after receiving service depart the system.

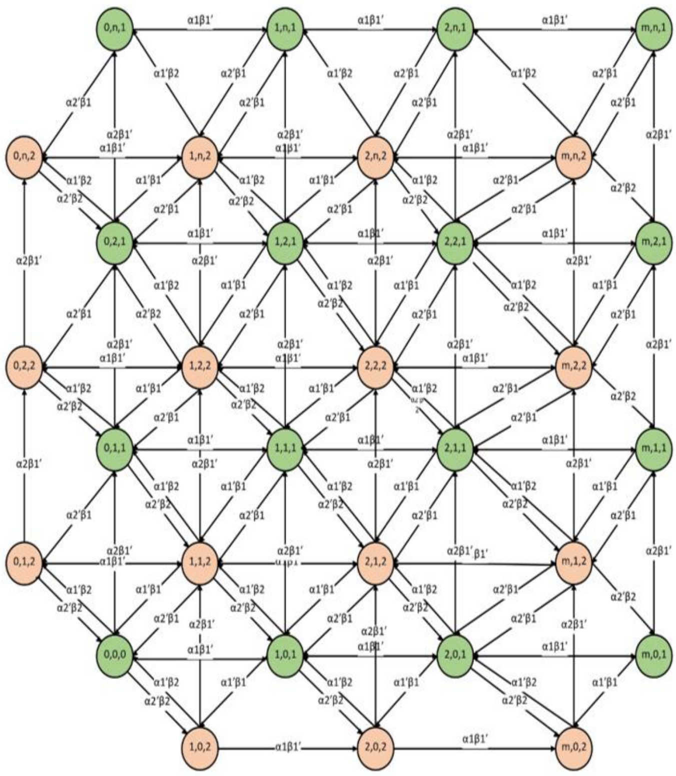

Figure 3: EAS Markov Chain

2.4 Flowprocess and Service Chain

The arrangement of servers that arrivals use while they are in the system are represented by the service chain (in this case, two series servers). In proposed system:

Arrivals exit the parallel queue and enter Server1 (First Stage), the first server in the series. Here, they are first processed or served.

Arrivals proceed to Server2 (Second Stage) if there is a second stage for additional processing or servicing.

Exit Point: Arrivals leave the system once all stages (servers) have been completed. The service chain of a customer's begins with the probability of β.

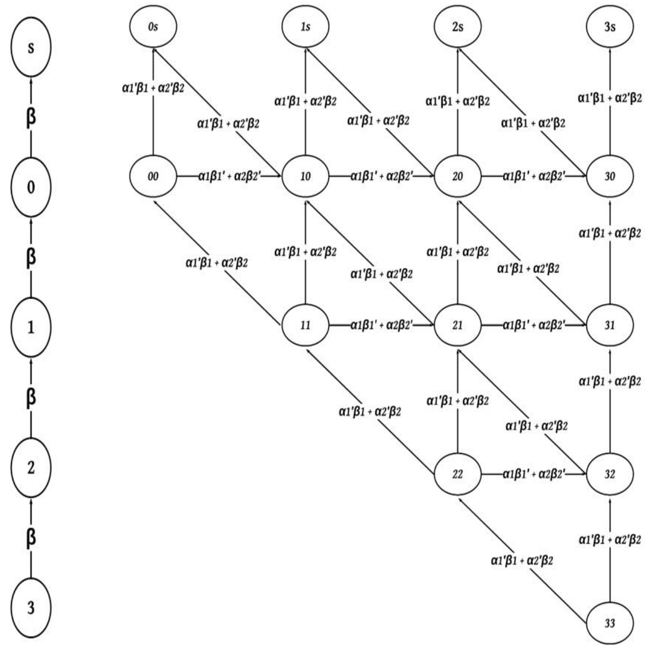

Figure 4: Service and Flowprocess chain

The entering of arrivals into the system determines a customer's flowprocess. The flowprocess chain illustrates the flow of each customer in the system graphically. It depicts all possible customer flow paths as well as the entering states where customers are permitted to begin the flow into the system and exit the system after receiving the service. It examines the complete flowprocess path of the customers, from when it enters the system to receive the service to when it exits the system. The system's flow process in figure 4, provides data on system state behavior and calculates the probability of customer events in any state.

Analytical equations:

These equation are derived from flowprocess markov chain to calculate probability mass function (σ) for each starting state (00, 11, 22, and 33) with stationary probability (π).

$$\sigma_{00} = \left(\frac{\pi_{00}\beta}{\pi_{00}+\pi_{11}+\pi_{22}+\pi_{33}}\right) \times \left(\frac{1}{1-\frac{\pi_{00}}{\pi_{00}+\pi_{11}+\pi_{22}+\pi_{33}}}\right) \quad (1)$$

$$\sigma_{11} = \left(\frac{\pi_{00}\beta'}{\pi_{00}+\pi_{11}+\pi_{22}+\pi_{33}} + \frac{\pi_{11}\beta}{\pi_{00}+\pi_{11}+\pi_{22}+\pi_{33}}\right) \times \left(\frac{1}{1-\frac{\pi_{11}}{\pi_{00}+\pi_{11}+\pi_{22}+\pi_{33}}}\right) \quad (2)$$

$$\sigma_{22} = \left(\frac{\pi_{11}\beta'}{\pi_{00}+\pi_{11}+\pi_{22}+\pi_{33}} + \frac{\pi_{22}\beta}{\pi_{00}+\pi_{11}+\pi_{22}+\pi_{33}}\right) \times \left(\frac{1}{1-\frac{\pi_{22}}{\pi_{00}+\pi_{11}+\pi_{22}+\pi_{33}}}\right) \quad (3)$$

$$\sigma_{33} = \left(\frac{\pi_{22}\beta'}{\pi_{00}+\pi_{11}+\pi_{22}+\pi_{33}} + \frac{\pi_{33}\beta}{\pi_{00}+\pi_{11}+\pi_{22}+\pi_{33}}\right) \times \left(\frac{1}{1-\frac{\pi_{33}}{\pi_{00}+\pi_{11}+\pi_{22}+\pi_{33}}}\right) \quad (4)$$

3. Results and Discussion

Results are written in MATLAB for analytical and simulation purpose for calculations of mean number, delay, idle probability using probability distributions (Hypergeometric and Hypogeometric distributions). Also probability mass function and flow time using markov and flowprocess chains.

The figures 5,6,7 and 8 display how mean number of customers in both Queue 1, Queue 2 individually and the overall system changes when the total arrival rate α changes from 0-5 with different service rates for each scenario.

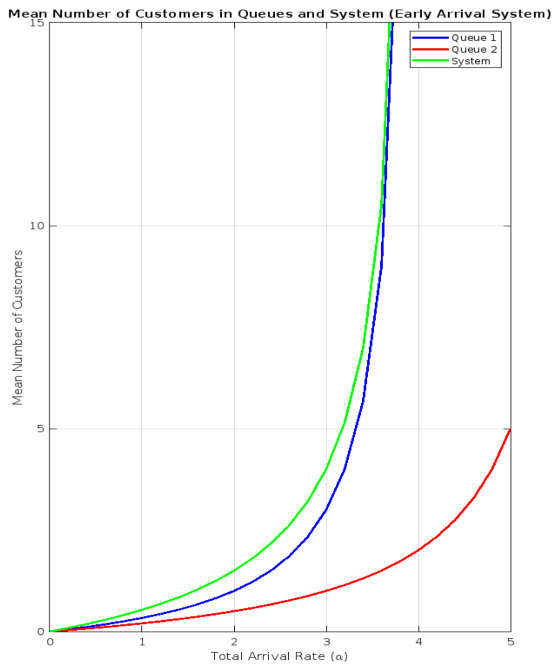

Figure 5: Scenario-I (β_1= 2 < β_2= 3, α= 0 to 5)

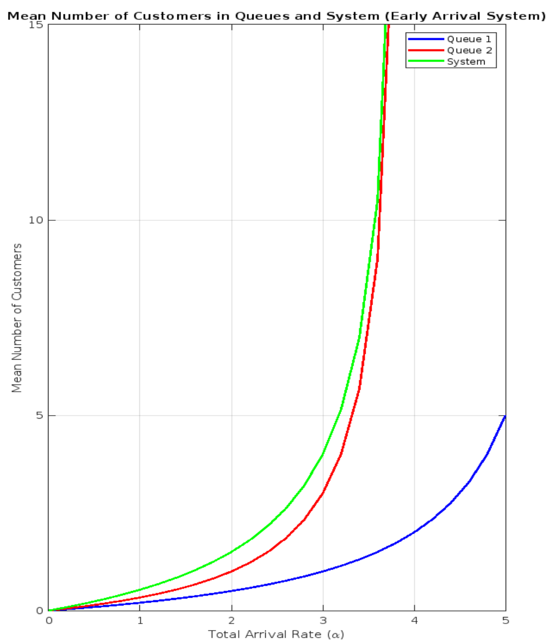

Figure 6: Scenario-II (β_1= 3 > β_2= 2, α= 0 to 5)

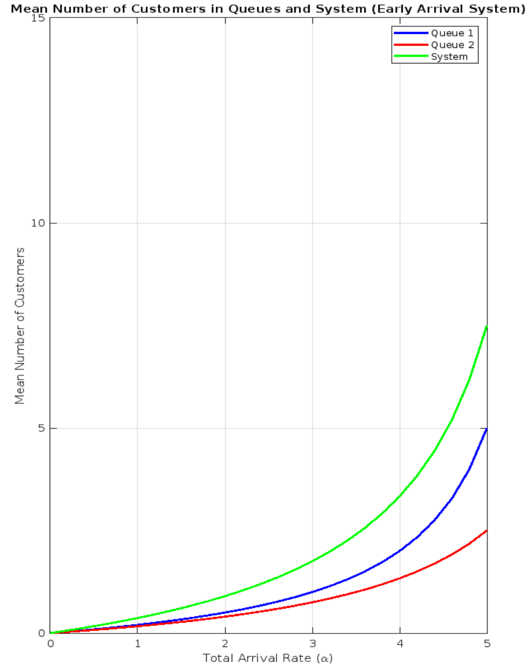

Figure 7: Scenario-III (β_1= 3 < β_2= 3.5, α= 0 to 5)

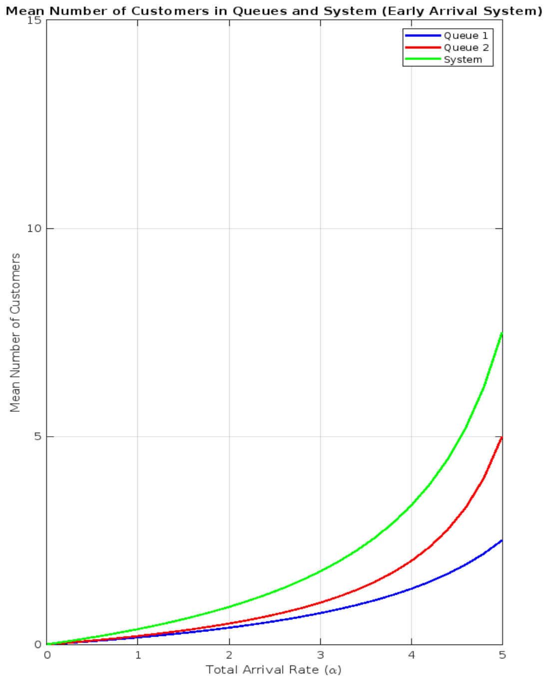

Figure 8: Scenario-I (β_1= 3.5 > β_2= 3, α= 0 to 5)

Figures 9, 10 and 11 below display idle and stationary probabilities, mean number and waiting time using probability distributions (Hypogeometric and Hypergeometric distribution) in both phases for α_1 = 0 to 1, α_2 = 0.3, β_1= 0.2 and β_2= 0.3.

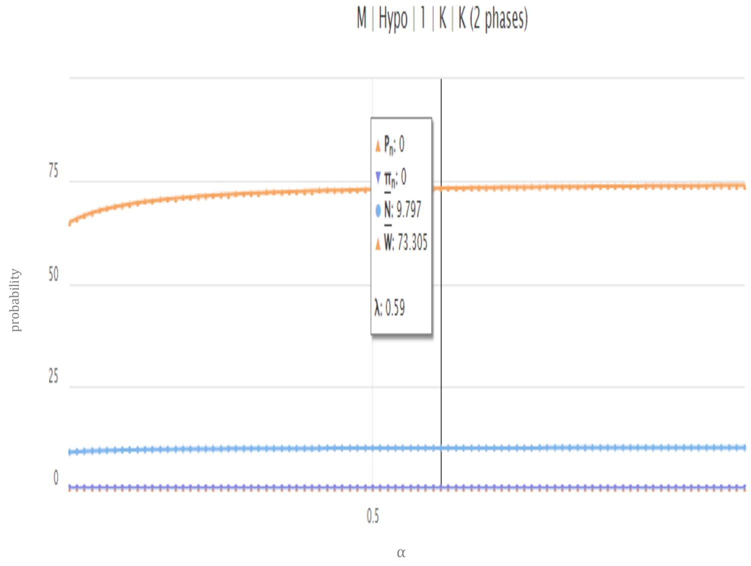

Figure 9: Idle Probability, Mean number in the system and Waiting / Delay time using Hypogeometrical distribution

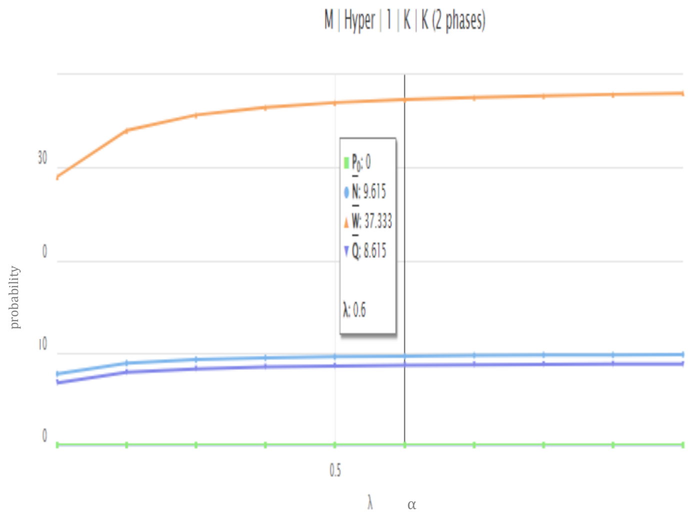

Figure 10: Idle Probability, Mean number in the system and Waiting / Delay time using Hypergeometric distribution

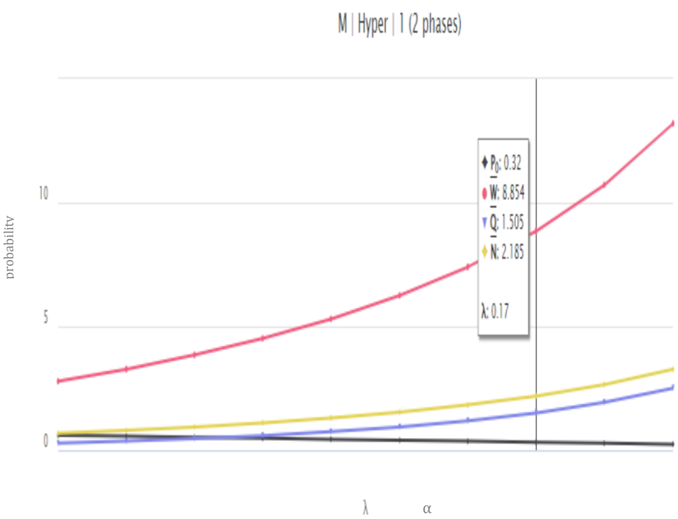

Figure 11: Hypergeometric distribution of DTQS

The following figures 12, 13, 14 and 15 are graphical representation of discrete queuing system models and program computes its stationary probabilities, simulates its progression through time, and visualizes the probability distributions at various time steps. The subplots show the pmf for each starting state, as well as the expected total flowtime. This comprehends the queuing system's behavior under the parameters such as arrival rate, service rate, and transition probabilities using different values for each scenario.

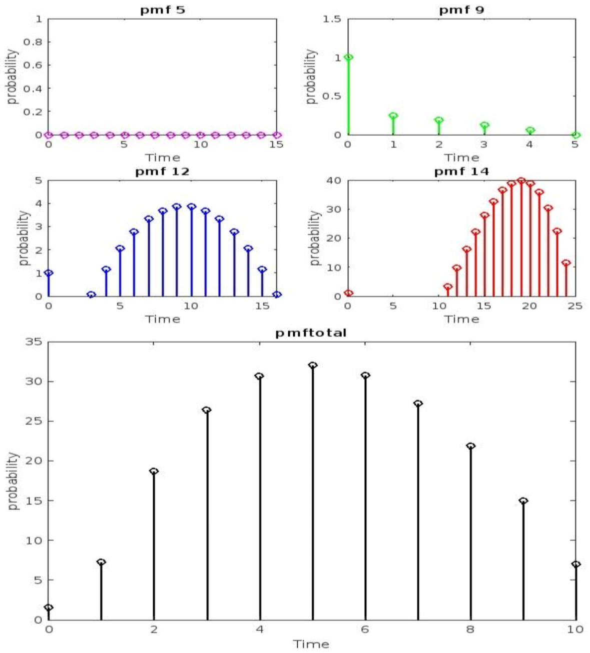

Figure 12: Probability mass function Scenario-I ($\alpha_1 < \alpha_2$, $\beta_1 < \beta_2$)

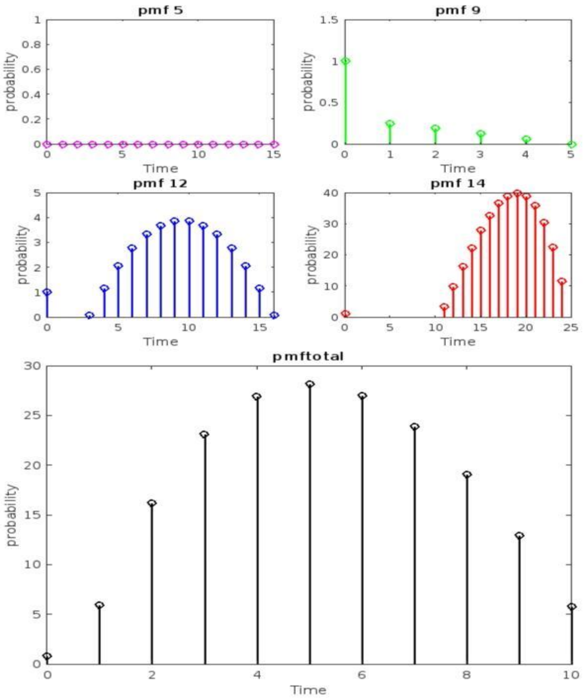

Figure 13: Probability mass function Scenario-II ($\alpha_1 > \alpha_2$, $\beta_1 > \beta_2$)

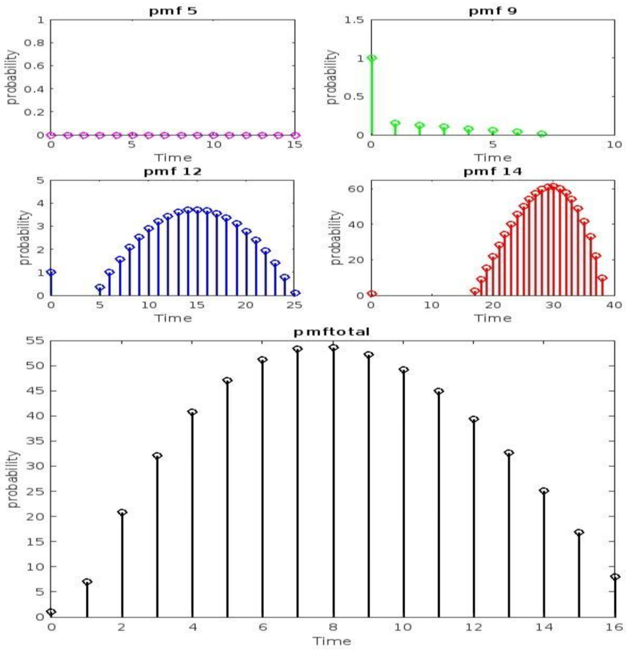

Figure 14: Probability mass function Scenario-III ($\alpha_1 > \alpha_2$, $\beta_1 < \beta_2$)

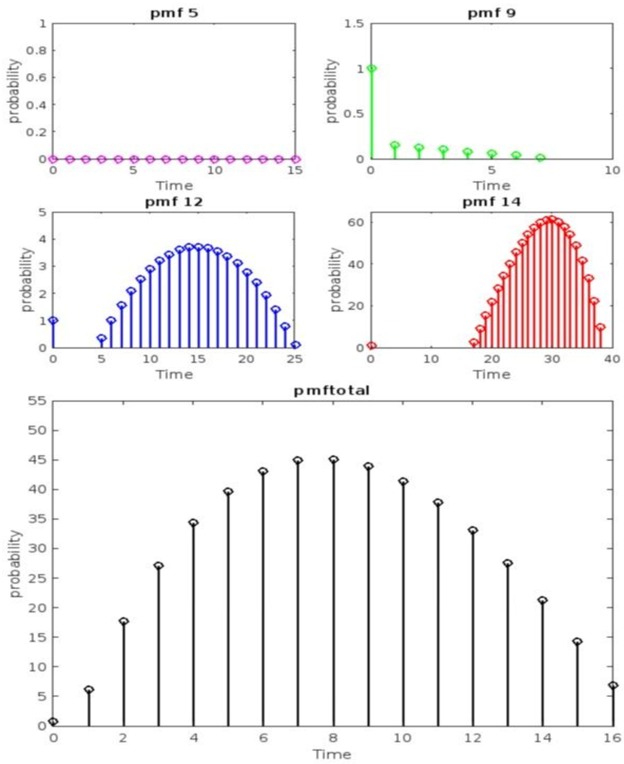

Figure 15: Probability mass function Scenario-I ($\alpha_1 < \alpha_2$, $\beta_1 > \beta_2$)

4. Conclusion

This paper aimed to analyze system congestion using a discrete-time system having variable thresholds to avoid system blocking. Performance analysis of the discrete-time queuing system evaluates its efficiency in managing arrivals, behavior at different system sizes, delays, flowtime, and probability mass function for each customer. Early arrival system's Markov chain, flowprocess, and service chains were developed to depict system behavior by utilizing variable thresholds to prevent blocking. The system utilizes discrete time queue with variable thresholds for early arrival, providing valuable insights into congestion management and performance. The study provides insight into simulations usage when analytical approaches get too complex, which can provide further insights into system behavior. A comparative analysis elucidates the quantitative and qualitative effects on early arrivals on system performance.

Lastly, this study not only increases our theoretical understanding of queuing systems but also provides suggestions for optimizing system performance in contexts with early arrivals. These findings have profound consequences for various applications, from service industries to network management, where optimal resource allocation is critical.

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