Performance Analysis of Open Tandem Queuing Network using Vector domain

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Abstract: In this paper, we tackle the problem of assessing an open tandem queuing system's flow time, which is sometimes challenging to examine in the conventional scalar domain. We suggest using the vector domain to analyze the flow time of an open tandem queuing network in order to get over this restriction. Additionally, we aim to develop a comprehensive service process chain for the open queueing network, which will provide a clear understanding of the flow of tasks and their completion times within the system. This will make it possible for us to precisely gauge and control network flow time The matrix geometric approach will be employed. to generate analytical equations in the vector domain in order to accomplish these goals. This method has been proven to be effective in analyzing complex queuing systems and provides a better representation of interconnected tasks in tandem. By analyzing the flow time in the vector domain and developing a service process chain, we expect to gain valuable insights into the behavior of open tandem queueing systems. This will have implications for various applications, such as improving the efficiency of service-oriented industries, optimizing resource allocation, and enhancing overall system performance. Overall, this research paper seeks to contribute to the understanding of flow time analysis in open tandem queueing systems. By moving from the scalar to the vector domain and applying the Matrix Geometric method, this study seeks to provide a more comprehensive and accurate analysis of the flow time dynamics and to facilitate the development of effective strategies for optimizing system performance in such complex queuing networks.

Keywords: *Open Tandem Queue, Flow process, Markov process, Service process, Probability density function.*

1. Introduction

The study of waiting lines through mathematics is known as queueing theory, which makes it possible to mathematically evaluate several related activities such a client entering the queue, waiting in the queue, and receiving service from a server. The input process, also known as the arrival process, the output process, also known as the service process, as well the system's capacity and user behavior, are the primary features of a queuing system.Tandem queues are one type of multistage service system that offers multiple stages of service. In such a system, the consumer has the option of continuing to get services in succession or quitting after only a short period of time. Tandem systems are used in communication networks to offer the services sequentially. When evaluating the effectiveness of broadband communication networks for network optimization, tandem queues are widely utilized. Broadband communication networks are used to deliver multimedia services, and these networks use a variety of service stages in succession to transmit diverse types of traffic, including audio, video, and data.

Tandem queues are the simplest open queuing system and multi-server system. In this method, a customer completes the service procedure by moving through a number of service stations in series [2]. Tandem queues with limited or infinite buffers in telecommunication networks have received a lot of attention. Following service, the served customer rejoins the second queue and a new customer arrives, joining the first line. This procedure was carried out repeatedly until the customer received the requested service. Tandem queues can be used to assess the network performance utilizing a variety of arrival and service processes.

2. Related Work

Gordan and Newell [10] and Hordijk and Van Dijk [6] propose exact answers for the intermediate infinite buffers of tandem queues as a state probability product form representation. Servers with finite buffer sizes and service rates with negative exponential distributions are studied by Labetoulle and Pujolle [8], Konheim and Reiser [9], and others. Takahashi [7] resolves the tandem queues system via an approximation method. A. Gomez [4] proposed the tandem queue with blocking in which no intervening buffer is required. Tandem queues with a single server and finite queues are discussed by Scott Spicer [3]. Dong-Won Seo [1] investigates a tandem queue with m nodes and discusses the deterministic service. Different analytical techniques have been used to evaluate the performance of the tandem queue system. One of the effective methods to study the tandem queue system is the Matrix Geometric method [5]. In order to calculate the system's performance metrics, the state probabilities of the tandem queue system are solved using the repeating structure of the vector state Markov chain. The repeated structure and boundary structure of the continuous time Markov chain allow us to solve the tandem queue system efficiently using the matrix geometric method. In networks, tandem queues are a transmission link that transmits traffic from one node to another. Tandem queues are formed by queues running sequentially. In this paper, we

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propose a three-queue parallel open tandem system with an analytical model, where the first stage output is the arrival of the second stage, the second stage output is the arrival (input) of the third stage, and so on. Using the Matrix Geometric Method and the vector domain method, the model is resolved.

3. Methodology

3.1 System model

In a tandem queuing network, multiple queues are linked in succession so that the output of one queue acts as the arrival rate for the following queue in the chain. In other words, clients proceed sequentially from one queue to the next. Each queue in a tandem queuing network has its own service rate and arrival rate. The capacity of the queue to serve clients is represented by the service rate, but the pace at which consumers join the queue is shown by the arrival rate. A network of queuing systems with two or more servers linked in series is known as an open tandem queuing network. In this network, clients or tasks are delivered to the first server and handled in turn. Once they have completed serving at the first server, and so forth until they leave the network. Key characteristics and factors to consider in an open tandem queuing network include:

•Arrival rate: The frequency of clients or tasks reaching the network's first server

•Service rate: The rate at which customers or tasks are processed at each server.

•Number of servers: The number of servers in the network as a whole. Wait times can be decreased by adding more servers, which can increase system efficiency..

•Service discipline: The method, such as first-come, first-served or priority-based scheduling, that determines how customers or work are handled at each server.

•Queue Lengths and waiting times: The total number of customers or tasks waiting in each line, how long they are there, and how long it takes before they are served.

•Throughput and system capacity: The maximum capacity of the overall system as well as the number of clients or jobs processed per unit of time.

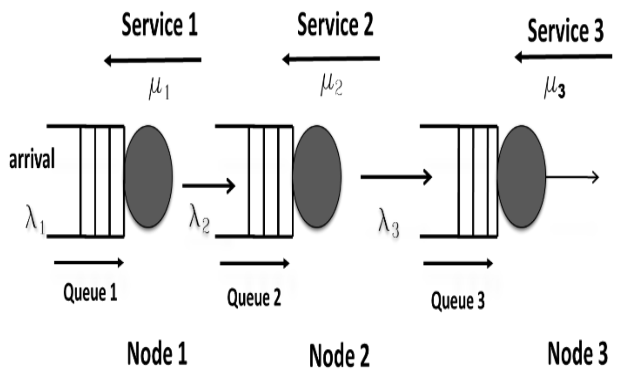

Figure.1. Open Tandem Queuing Network

3.2 Markov Chain

In different contexts a mathematical stochastic tool for comprehending and assessing how to deal with a "queue" of customers. Internal flow process, service procedures, quantity of servers, queuing capacity, range of customers, and queuing discipline are all characteristics. a quantitative probabilistic tool for comprehending and deciding how to manage a "queue" of customers in various conditions. This facilitates the description as well as the evaluation of the internal behavior of the system. The flow process chain of the system, Markov, specifies that the future state of the system will only depend on the current state of the system, implying that we can predict the future state of the system based solely on the current state and without the need to consult or examine the system's entire history.

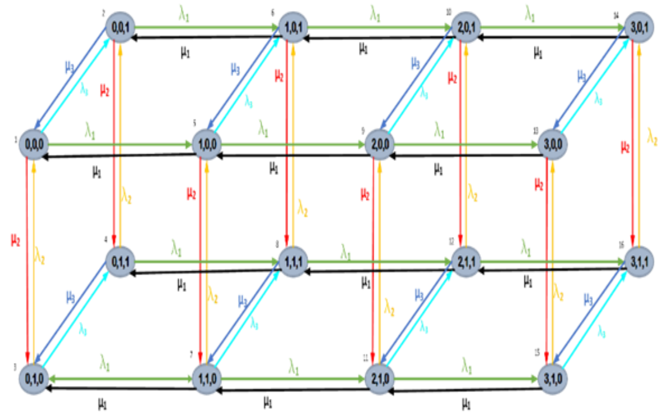

Figure.2. Markov Chain

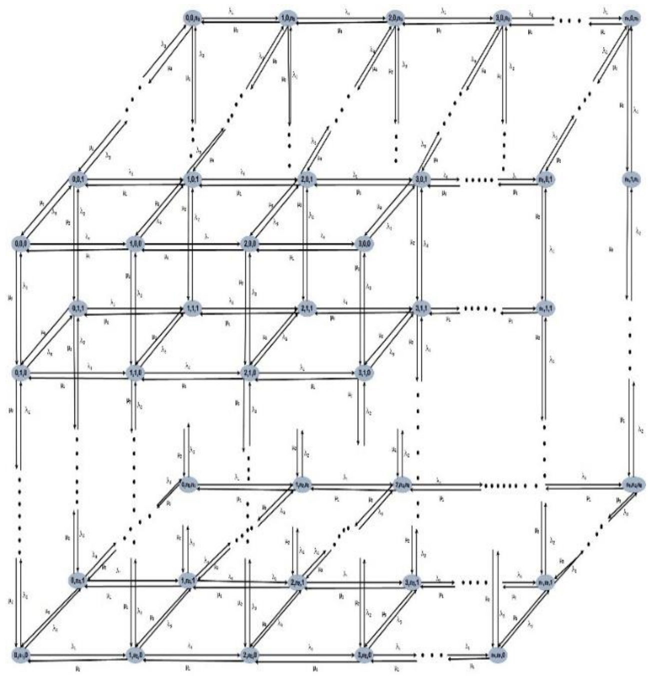

Figure.3. Generalized Markov Chain

State transition matrix of the above system is calculated as:

$$A0 = \begin{bmatrix} 0 & 0 & \lambda 2 \\ 0 & 0 & \lambda 3 \\ \mu 2 & \mu 3 & 0 \end{bmatrix}$$

$$A1 = \begin{bmatrix} 0 & 0 & 0 \\ \mu 1 & 0 & 0 \\ 0 & \mu 1 & 0 \\ 0 & 0 & \mu 1 \end{bmatrix}$$

$$A2 = \begin{bmatrix} 0 & 0 & \lambda 2 \\ 0 & 0 & \lambda 3 \\ \mu 2 & \mu 3 & 0 \end{bmatrix}$$

3.3 Matrix Geometric Method(MGM)

For resolving the stationary state probabilities for the Markov process with repeating structure, the Matrix Geometric Method (MGM) can be utilized [6]. The Matrix Geometric Method (MGM) divides the continuous time Markov chain (CTMC) into two halves: The starting (or boundary) levels: Its structure can be uneven, but it must be limited. Recurring (or repeating) levels: It has a regular structure despite being infinite. The CTMC's generating matrix Q is constructed lexicographically, with its components separated into finite submatrices [6]. Each submatrix indicates the transition rates of a distinct class of transitions.

Infinitesimal generator matrix of the system can be constructed as:

$$P = \begin{bmatrix}
\lambda' & \lambda_3 & \mu_2 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mu_3 & 0 & 0 & \mu_2 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_2 & 0 & 0 & \lambda_3 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_2 & \mu_3 & 0 & 0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mu_1 & 0 & 0 & 0 & 0 & \lambda_3 & \mu_2 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mu_1 & 0 & 0 & \mu_3 & 0 & 0 & \mu_2 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mu_1 & 0 & \lambda_2 & 0 & 0 & \lambda_3 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mu_1 & 0 & \lambda_2 & \mu_3 & 0 & 0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \mu_1 & 0 & 0 & 0 & 0 & \lambda_3 & \mu_2 & 0 & \lambda_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mu_1 & 0 & 0 & \mu_3 & 0 & 0 & \mu_2 & 0 & \lambda_1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \mu_1 & 0 & \lambda_2 & 0 & 0 & \lambda_3 & 0 & 0 & \lambda_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_1 & 0 & \lambda_2 & \mu_3 & 0 & 0 & 0 & 0 & \lambda_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_1 & 0 & 0 & 0 & 0 & \lambda_2 & \mu_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_1 & 0 & 0 & \mu_3 & 0 & 0 & \mu_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_1 & 0 & \lambda_2 & 0 & 0 & \lambda_3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_1 & 0 & \lambda_2 & \mu_3 & 0
\end{bmatrix}$$

The boundary portion's submatrices are identified by the symbol B:

- B_{00}: transitions inside level 0.

$$B_{00} = [\lambda']$$

- B_{01}: transitions from level 0 to 1.

$$B_{01} = [\lambda_3 \quad \mu_2 \quad 0]$$

- B_{10}: transitions from level 1 to 0.

$$B_{10} = \begin{bmatrix} \mu_3 \\ \lambda_2 \\ 0 \end{bmatrix}$$

The repeating portion's submatrices are identified by the A:

- A0 : transitions from current level to next level.

$$A0 = \begin{bmatrix} 0 & 0 & \mu_2 \\ 0 & 0 & \lambda_3 \\ \lambda_2 & \mu_3 & 0 \end{bmatrix}$$

- A1: transitions within level.

$$A_1 = \begin{bmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_1 & 0 \\ 0 & 0 & \mu_1 \end{bmatrix}$$

- Ak: transitions from current level to back level.

$$A_2 = \begin{bmatrix} 0 & 0 & \mu_2 \\ 0 & 0 & \lambda_3 \\ \lambda_2 & \mu_3 & 0 \end{bmatrix}$$

3.4 Flow and Service Process

The flow process graphically depicts each customer's movement through the system. The flow process depicts all conceivable customer flow pathways as well as entry states wherein customers may enter and exit the system after receiving the service. When a test data packet traverses a system that obtains a service, the complete flow process path is examined. The system's flow process analyses the behavior of system states and computes the likelihood that a customer would arrive in any state other than the state of blockage. The current state of the system and the flow of services. Markov chains are used to build the flow process Markov chain.

1) The flow process begins when a customer enters the system. The starting states are those conditions.

2) The flow process does not start in the current conditions. The term "secondary states" refers to these.

3) The condition under which the flow procedure concludes. The term "absorbing states" refers to these.

The flow process starts when a customer enters the system and is acknowledged. Only three structure conditions can accept the trial data collection in the saturated system..

States 10 and 21 are the system's initial states, where the consumer first joins the system at node 1 as shown in figure 4 and states 10 and 21 are the system's initial states, where the customer join the system at node 2 as shown in figure 5 and states 10 and 21 are the initial states, where the customer join the system at node 3 as shown in figure 6.

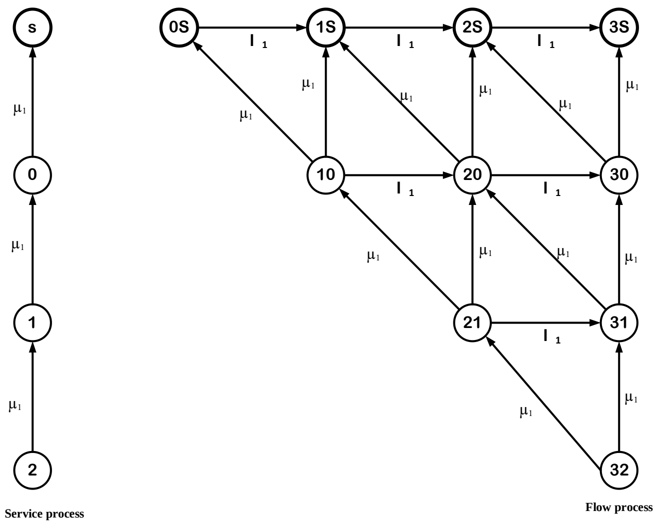

Figure.4. Flow process and service process of tandem Node1

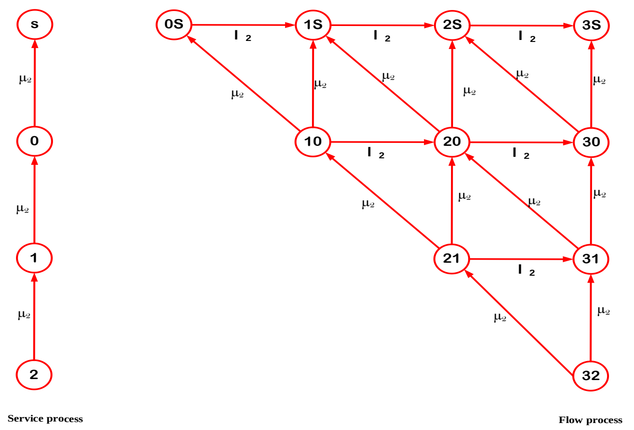

Figure. 5 Flow process and service process of tandem Node 2

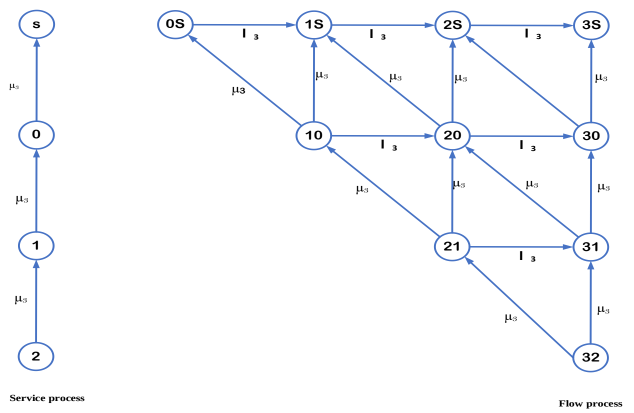

Figure. 6 Flow process and service process of tandem Node 3

Overall system behavior of Markov chain through flow process is represents in figure 7. This shows the all system flow process and service process.

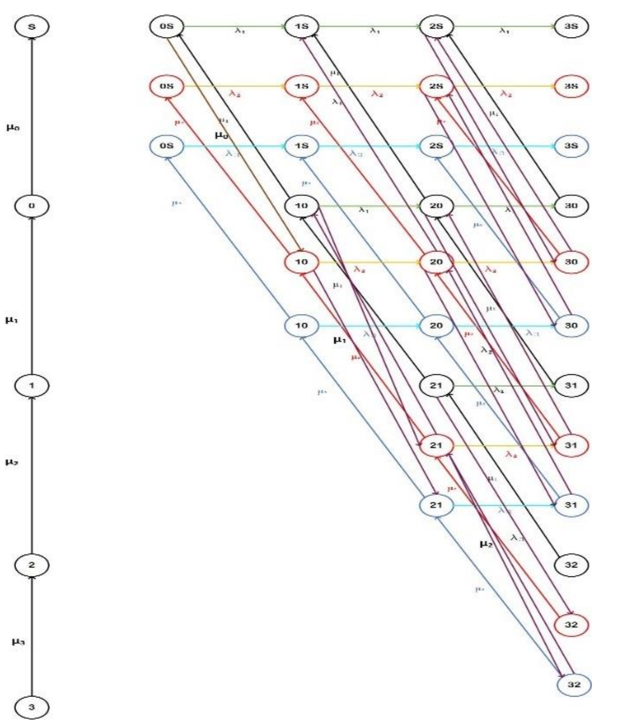

Figure.7 Flow process and service process of overall system

Flow process system entry pdf equations

At node 1

$$\sigma'_{10} = \frac{\pi'_{10}}{\pi'_{10} + \pi'_{21}} * \frac{1}{1 - \frac{\pi'_{10}}{\pi'_{10}+\pi'_{21}}}$$

$$\sigma'_{21} = \frac{\pi'_{21}}{\pi'_{10} + \pi'_{21}} * \frac{1}{1 - \frac{\pi'_{21}}{\pi'_{10}+\pi'_{21}}}$$

At node 2

$$\sigma''_{10} = \frac{\pi''_{10}}{\pi''_{10} + \pi''_{21}} * \frac{1}{1 - \frac{\pi''_{10}}{\pi''_{10}+\pi''_{21}}}$$

$$\sigma''_{21} = \frac{\pi''_{21}}{\pi''_{10} + \pi''_{21}} * \frac{1}{1 - \frac{\pi''_{21}}{\pi''_{10}+\pi''_{21}}}$$

At node 3

$$\sigma'''_{10} = \frac{\pi'''_{10}}{\pi'''_{10}+\pi'''_{21}} * \frac{1}{1-\frac{\pi'''_{10}}{\pi'''_{10}+\pi'''_{21}}}$$

$$\sigma'''_{21} = \frac{\pi'''_{21}}{\pi'''_{10} + \pi'''_{21}} * \frac{1}{1 - \frac{\pi'''_{21}}{\pi'''_{10}+\pi'''_{21}}}$$

Overall Equation

$$\sigma_{10} = \left(\frac{\pi'_{10}}{\pi'_{10}+\pi'_{21}} * \frac{1}{1-\frac{\pi'_{10}}{\pi'_{10}+\pi'_{21}}}\right) + \left(\frac{\pi''_{10}}{\pi''_{10}+\pi''_{21}} * \frac{1}{1-\frac{\pi''_{10}}{\pi''_{10}+\pi''_{21}}}\right) + \left(\frac{\pi'''_{10}}{\pi'''_{10}+\pi'''_{21}} * \frac{1}{1-\frac{\pi'''_{10}}{\pi'''_{10}+\pi'''_{21}}}\right)$$

$$\sigma_{21} = \left(\frac{\pi'_{21}}{\pi'_{10}+\pi'_{21}} * \frac{1}{1-\frac{\pi'_{21}}{\pi'_{10}+\pi'_{21}}}\right) + \left(\frac{\pi''_{21}}{\pi''_{10}+\pi''_{21}} * \frac{1}{1-\frac{\pi''_{21}}{\pi''_{10}+\pi''_{21}}}\right) + \left(\frac{\pi'''_{21}}{\pi'''_{10}+\pi'''_{21}} * \frac{1}{1-\frac{\pi'''_{21}}{\pi'''_{10}+\pi'''_{21}}}\right)$$

4. Results and Discussion

MATLAB programs are built for analytical and simulation purposes in order to calculate the probability density function along with flow times. The flow procedure calculates the likelihood of a customer appearing in any situation except the blocking state. The probability density function at node 1is shown in figure 8 , the probability density function at node 2 is shown in figure 9 and the probability density function at node 3 is shown in figure 10

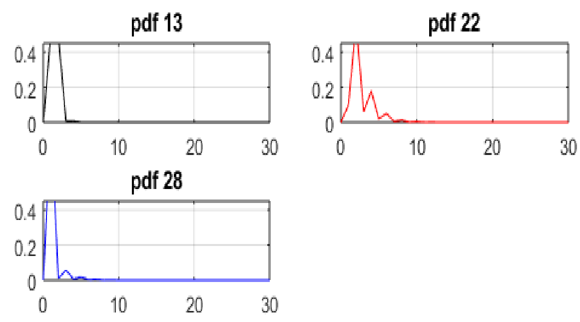

Figure.8 Probability Density Function at Node 1

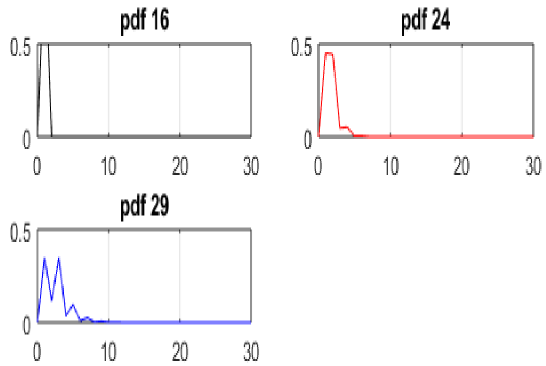

Figure . 9 Probability Density Function at Node 2

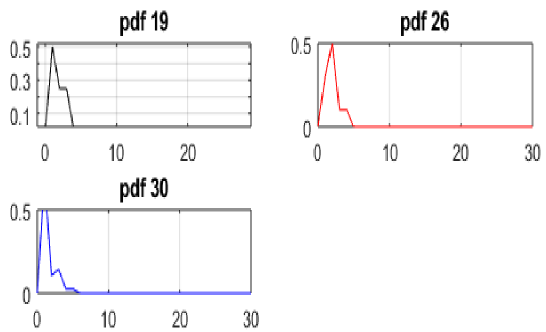

Figure. 10 Probability Density Function at Node 3

Results are also generated from queuing system assistance software for calculation of System Intensity, System Utilization, Idle Probability, Mean number in the system, Waiting Time, Mean number in the queue.

Scenario 1:When arrival λ=0 to 1 so service rate μ=0.3 S =10. figure 11, illustrate the scenario 1

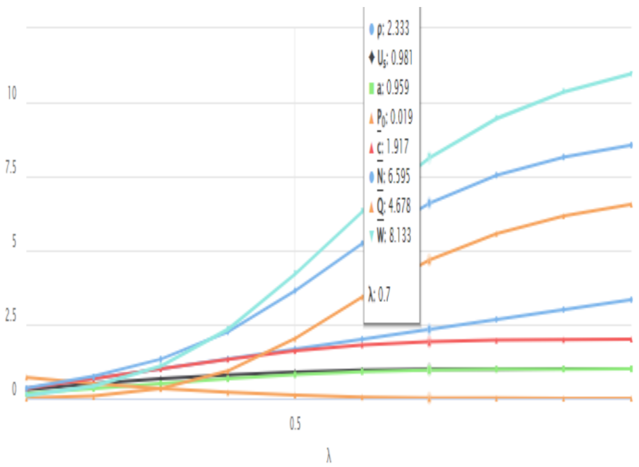

Figure. 11 Scenario 1: arrival λ=0 to 1 so service rate μ=0.3

Scenario2 : When arrival λ=0 to 1 so service rate μ =0.5 S =25. figure 12 illustrate the scenario 2

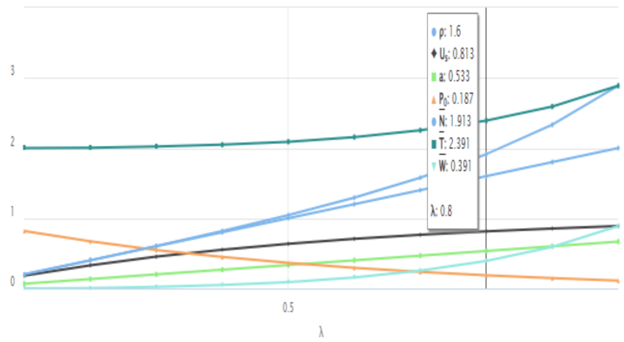

Figure. 12 Scenario2 : arrival λ=0 to 1 so service rate μ =0.5.

Scenario 3 :When arrival λ=0 to 1 so service rate μ =0. 1 S =05. Figure 13 illustrate the scenario 3.

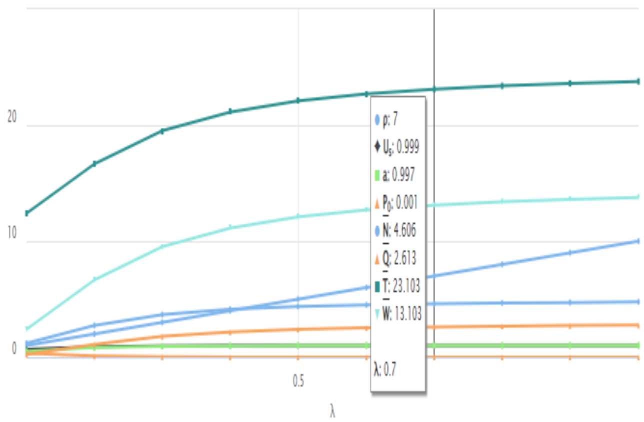

Figure. 13 Scenario 3 : arrival λ=0 to 1 so service rate μ =0. 1

5. Conclusion

This paper discusses the service and flow process analysis of a queuing system. Process The transition matrix of a Markov chain was calculated. The starting states were represented using a Markov flow technique. The system's starting state probabilities were computed. The simulation findings from analytical and simulation results were used to demonstrate the probability density function of starting states and the total probability density function.

Open tandem queueing network is analysed for the system optimization. System is analysed by two techniques Matrix Geometric method(Vector Domain) and Flow Process. It provides overall system behaviour information.

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