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Discrete Time Slot Analysis of a Non Preemptive Priority Queuing System Using Early Arrival Technique

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Abstract: In this paper, we take into account a discrete-time non-preemptive priority queuing system having two servers and two queues. Rate of arrival and rate of service is geometrically distributed. We use early arrival approach to analyze system. Appling this method a Markov chain for the system is built that defines relationships between the state probabilities of the system. These probabilities are used to derive analytical equations of system. We generate and calculate a number of system performance parameters, including system server utilization, throughput, and system size by using analytical equations obtained from Markov chain in MATLAB. For the purpose of validation of obtained results we then used MATLAB Simulink.

Keywords: *Multi-servers, Multi-queue, Non-preemptive priority system, Queueing System*.

1. Introduction

Queueing theory is mathematical model of explaining waiting queue. In simple terms, it is the study of queues, or waiting lines. In general, a queueing system exists whenever "customers" want "service" from a facility; both the customers' arrival and the times for providing service are supposed to be random. New customers will often wait in queue for the next server that becomes available if all of the 'servers' are already full when they arrive. Queuing theory has many examples that include the queues at ticket booths, theatres, bank tellers, train booking offices, check stands at supermarkets, and clinics and doctor's offices [1-6].

One of discipline of serving customers is by priority scheduling. Wherein customers are labelled and served according to the priority scheme: high-priority tasks preempt low-priority tasks in the queue. Preemptive priority which allows service interruption and non-preemptive priority which bans service interruption. Numerous applications use priority queueing [7-9]. In terms of queueing, performance measures are frequently utilized. A significant portion of queueing theory is devoted to the analysis of priority queues [10-18]. For this, performance metrics of priority system are measured.

2. Related Work

The study of single class discrete time queueing systems has produced a large body of literature. The focus of Rubin and Tsai's [12] work on discrete-time non-preemptive queues is on the mean waiting time for a discrete-time queue fed by an i.i.d. arrival mechanism. The early research on discrete-time multiserver queues with single arrivals was done by the authors of [13], who also discussed the GI/Geom/m queue with EAS. Chaudhry [14] did a thorough analysis of a multiserver discrete-time queue with batch arrivals for the early arrival system. EAS was used to address the GIX/Geom/m queue, and linkages between state probabilities at various epochs were created. chan [19] was first to use EAS technique applied on single server. Gupta et al. [14] used the supplementary variable technique and difference equation approach to establish the steady-state queue-length distribution at pre-arrival and arbitrary epochs for a discrete time GIX/GeoY/1 queue with an early arrival system (EAS). [15-16] have conducted a thorough investigation of the non-preemptive queues. Additionally taken into account by the authors were multi-server non-preemptive priority systems with Markovian arrival process.

Numerous studies on analyzing the system performance of priority multiserver queues have been conducted during the last ten years. [Pandey 17] has given a performance analysis of a discrete-time non-preemptive priority queue. Unlike the system and methods that are accessible and are described in the literature. We proposed performance analysis of discrete time non-preemptive priority queueing system by using early arrival techniques [EAS].

The paper is organized as follows: Section II defines methodology that describes the proposed system model, EAS, discrete-time Markov chain and then analytical equations of the system are discussed. Results from the simulation are presented in Section III. A conclusion is provided in Section IV.

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3. Methodology

In this research a discrete time non preemptive priority queueing model is developed having many servers and queues and then Markov chain of model is built by using technique of early arrival (EAS). By applying this technique, state transitions and probabilities of system are obtained. By solving, we get analytical equations of system and performance metrics are calculated by using these equations in MATLAB programs and simulation graphs are obtained. In order to validate results, Simulink software is utilized. We start from first describing model, EAS technique then developing system model, Markov chain, analytical equations, and results and in last validation of results.

3.1 Model Description

The discrete time non-preemptive priority queueing model is developed that is based on the aforementioned premises:

1. We considered a discrete-time queuing system where the time axis has been divided into fixed-length continuous periods called slots.
2. A geometric process with the arrival rate (α) parameter governs how customers enter the system. The likelihood that customers may join the system before a slot is available is taken into account.
3. The system contains C servers.
4. The system is S in size.
5. A geometric distribution with parameter (β) describes how long it takes for customers to be served.
6. Customers are served non-preemptively.
7. Only at a slot border may a consumer get service.

3.2 Early Arrival Technique (EAS)

In the modelling of systems in which the packets (information) must be transferred in the exact same slot in which it has arrived, discrete-time queues with EAS strategy are more important. In early arrival system, every arriving customer may start using the system in the slot where it arrives if the server is not already busy; in that situation, it awaits and queues in buffer, lengthening the line with each additional customer until the system is completely filled. When a new client arrives and the system is already full, even with just one customer in the server, entry will not be permitted. Only the s-1 state of the system is available to any new customers. When a client is served and no more customers are scheduled to come during that period, the queue's length shortens. If there is no arrival or departure from the system, or if there is just one arrival or departure within the busy state of the system (excluding idle and blocking state), the status of the system remains unchanged. When a system is in an idle state, nothing arrives and it is completely empty. When the system is in a blocking condition, no arrival is permitted to enter until the service is taken.

3.3 Proposed system model

In this research, a discrete time geometric non preemptive priority queuing model is developed that has two servers S1 and S2, along with two queues, Q1 and Q2 as shown in figure 1.

The system under discussion comprises two different kinds of arrivals ($\alpha1$ & $\alpha2$). S is the system's capacity. To serve the arrivals in queues, there are two different servers in the system. According to non-preemptive priority given to each work, β_1 serves the arrivals of both lines, while 2 does the same. Any sort of consumer arriving according to non-preemptive priority can be served by any server. Taking into account that customer $\alpha1$ is a high priority and customer $\alpha2$ is a low priority. The number of servers affects the system's performance.

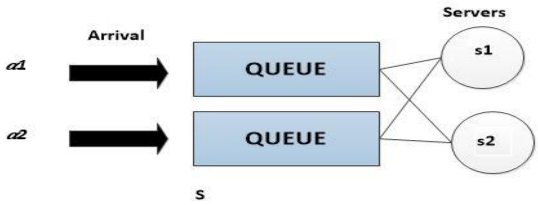

Figure.1. Non preemptive priority queuing model

3.3.1 Markov chain of proposed system

It is a mathematical process that switches from one state to another. System states, transitions and behavior of customers entering and leaving is determined by this.

The system's state transitions and probabilities are shown in Figure 2. There are two servers and two queues in the system. Two variables (i, j) are used to denote each state, with i standing for Q1 and j for Q2 (two queues). The system state (0, 0) denotes that there are no customers waiting to be served and that both queues are empty. After a client enters the system in Q1, the state changes from (0,0) to (1,0) as indicated by an arrow, and after the consumer receives service, the state decreases as indicated by a backward arrow. Similar to this, the system state changed from 0,0 to 0,1 when the client arrived in Q2, and it then declined as the consumer received assistance. Both queues are fully occupied and blocked in the state (S,S).

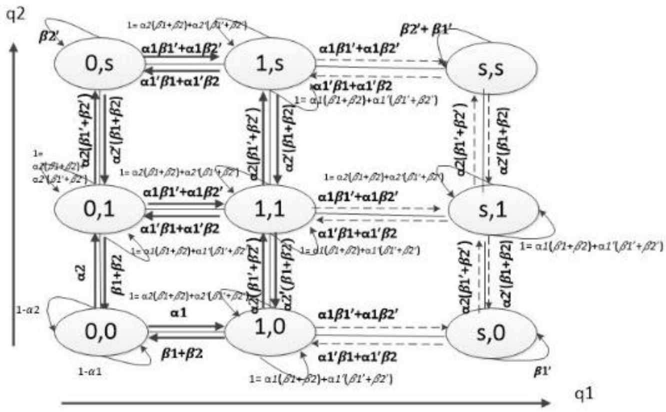

Figure.2. Markov chain of non-pre-emptive priority system

3.3.2 Analytical Equations of System

For Analytical equations, divide above Markov chain into three conditions, idle, busy and blocking.

State having no arrival and departure is idle state (0,0). Deriving equations from all incoming and outgoing probabilities from idle state as shown in figure 3. We got following equations.

For state (0,0).

$(\beta_1+\beta_2)\pi_{1,0} + (\beta_1+\beta_2)\pi_{0,1} + (1-\alpha_1)\pi_{0,0} + (1-\alpha_2)\pi_{0,0} = (1-\alpha_1)\pi_{0,0} + (1-\alpha_2)\pi_{0,0} + (\alpha_1)\,\pi_{1,0} + (\alpha_2)\,\pi_{0,1}$
(1)

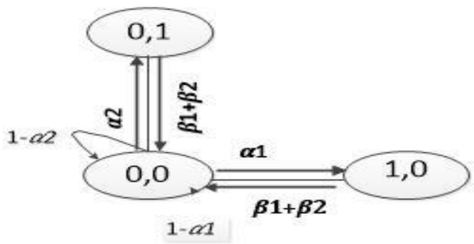

Figure.3. Idle state

Similarly, for busy States as shown in figure 4

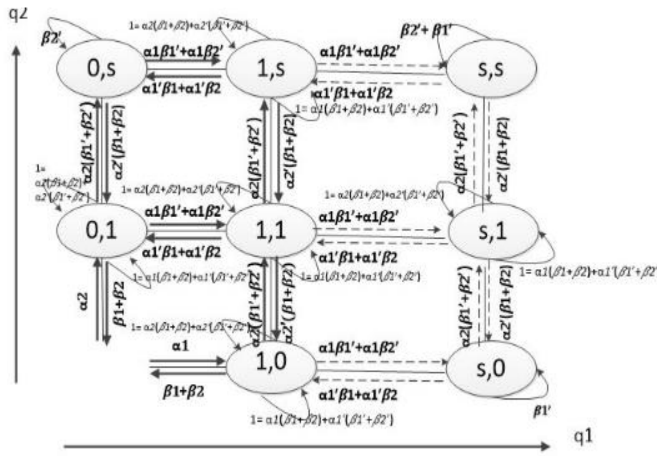

Figure.4. Busy states

For State (0,1)

$(\alpha_2)\pi_{0,0}+(\alpha_2'\beta_1+\alpha_2'\beta_2)\,\pi_{0,2} + (\alpha_1'\beta_1+\alpha_1'\beta_2)\,\pi_{1,1} + (\alpha_2(\beta_1+\beta_2)+\alpha_2'(\beta_1'+\beta_2'))\,\pi_{0,1} = (\alpha_2(\beta_1+\beta_2)+\alpha_2'(\beta_1'+\beta_2'))\,\pi_{0,1}+\beta_1+\beta_2(\pi_{0,0})+(\alpha_1\beta_1'+\alpha_1\beta_2')\,\pi_{1,1}+(\alpha_2\beta_1'+\alpha_2\beta_2')\,\pi_{0,2}$
(2)

Similarly State (0,s)

$\beta_2'\pi_{0,s}+\alpha_2(\beta_1'+\beta_2')\quad \pi_{0,s-1} + \quad (\alpha_1'\beta_1+\alpha_1'\beta_2)\quad \pi_{1,s} = \beta_2'\pi_{0,s}+\alpha_2'\beta_1+\alpha_2'\beta_2)\quad \pi_{0,s-1} + \quad \alpha_1(\beta_1'+\beta_2')\quad \pi_{1,s}$
(3)

State (s,s)

$(\beta_1' + \beta_2')\pi_{s,s} + (\alpha_2\beta_1'+\alpha_2\beta_1')\quad \pi_{s,s-1} + (\alpha_1\beta_1'+\alpha_1\beta_2')\ \pi_{s-1,s} = \quad (\beta_1' + \beta_2')\pi_{s,s}+\alpha_1'\beta_1+\alpha_1'\beta_2)\quad \pi_{s-1,s} + \alpha_2'\beta_1+\alpha_2'\beta_2)\,\pi_{s,s-1}$ (4)

Similarly, for blocking State as shown in figure 5

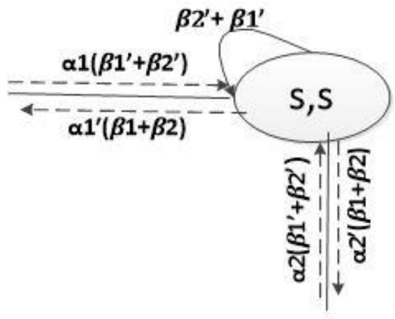

Figure.5. Blocking state

State (s,s)

$(\beta_1' + \beta_2')\pi_{s,s} + (\alpha_2\beta_1'+\alpha_2\beta_1')\pi_{s,s-1} + (\alpha_1\beta_1'+\alpha_1\beta_2')\ \pi_{s-1,s} = (\beta_1' + \beta_2')\pi_{s,s}+\alpha_1'\beta_1+\alpha_1'\beta_2)\pi_{s-1,s} + \alpha_2'\beta_1+\alpha_2'\beta_2)\,\pi_{s,s-1}$ (5)

4. Results and Discussion

Measurements of queue length, waiting time, server utilization, and other metrics are used to evaluate system performance. These measurements are computed using MATLAB simulations. Where using formulas derived from of the analytical equations discussed in this study programs are created. They are then appropriately examined, and simulation outcomes are discovered. In our work, we employed a variety of alpha and beta values for the analysis. Alpha represents the average number of arrivals per unit of wait time, whereas beta represents the system's average service rate. As a consequence, given the specified values of alpha and beta, we have obtained various outcomes for system. Simulation graphs provide system performance metrics and for validation we use MATLAB Simulink where same model is created and analyzed for same value of alpha and beta values.

4.1 MATLAB Simulation

We got these below performance graphs by writing programs in MATLAB using above analytical equations discussed in section 3.3.2. In graphs we have taken x-axis and y-axis where x-axis denotes alpha values starting from 0.1 to 0.9 and y–axis can be metric (queue length, server utilization and waiting time (delay time) and on three beta values 0.6, 0.7 and 0.8 values system is analyzed. Figure 6 represents average number of customers (Queue length), figure 7 represents average number of customers in system (system length) ,figure 8 represents waiting time in queue (queue delay time) , figure 9 represents waiting time in system (system delay time) and figure 10 represents server utilization.

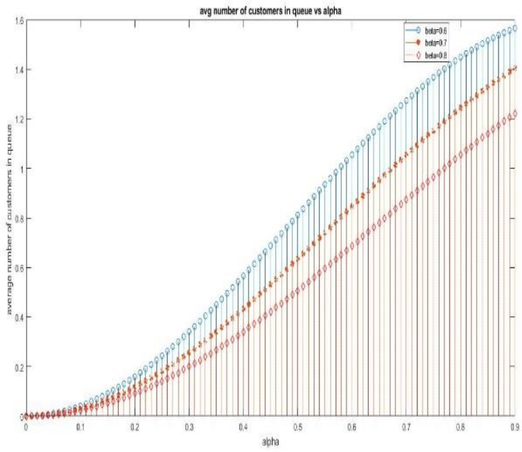

Figure.6. Average number of customers in queue (queue length)

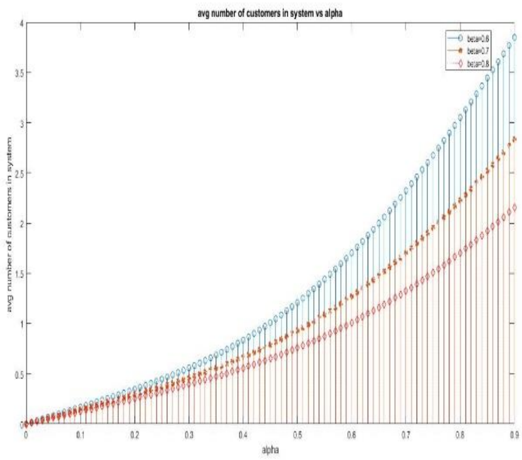

Figure. 7. Average number of customers in system (system length)

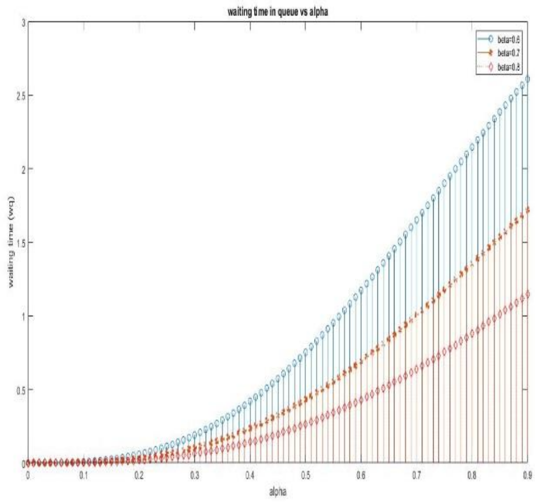

Figure. 8. Waiting time in queue (queue delay time)

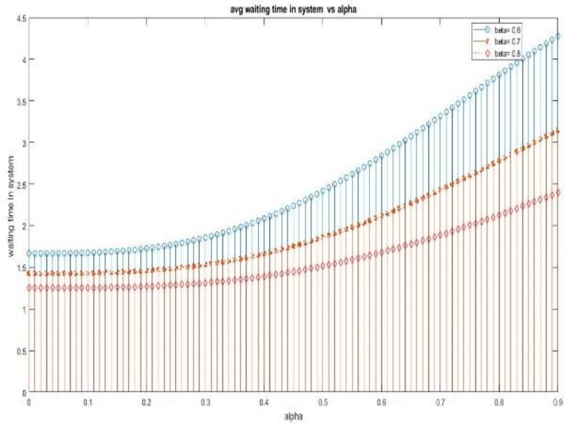

Figure. 9. Waiting time in system (system delay time)

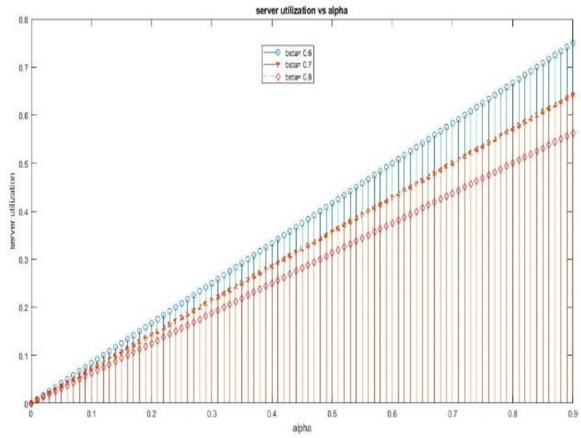

Figure. 10. Server utilization

4.2 Validation

The ultimate goal of process validation is to show, with a high degree of certainty, that the process can generate goods that can be made consistently while satisfying preset standards within given constraints.

We use MATLAB Simulink to validate our non-preemptive priority model. We use same arrival and service rates, non-preemptive queues, servers and have got same response by analyzing system.

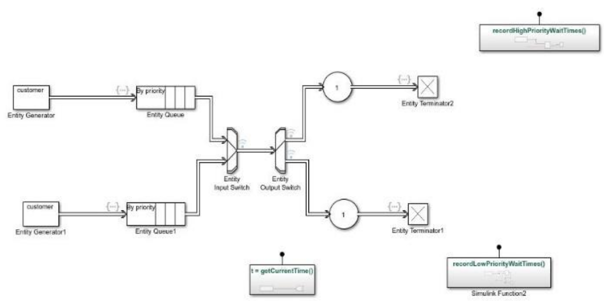

Figure. 11. Simulink model

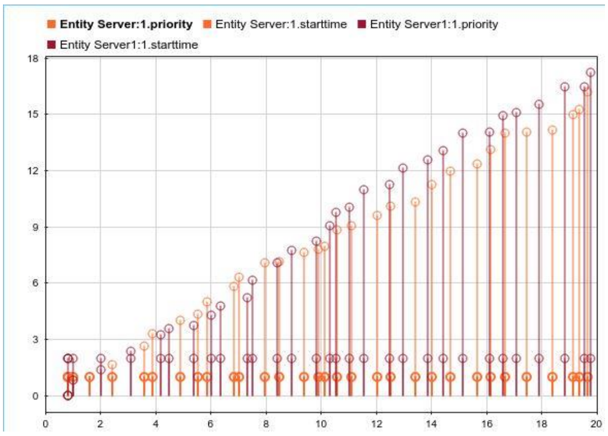

Figure. 12. Simulink graph

5. Conclusion

In this paper, a queuing system with non-preemptive priority scheduling was the subject of our analysis. Performance measures i.e. queue length, delay time, server utilization are calculated and analyzed on three different values and same parameters are analyzed on Simulink software. This analyses of system gives overall performance of system.

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